The Cost of Adaptivity in Security Games on Graphs

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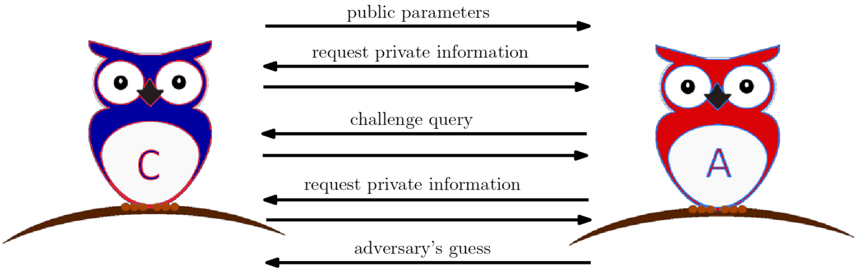
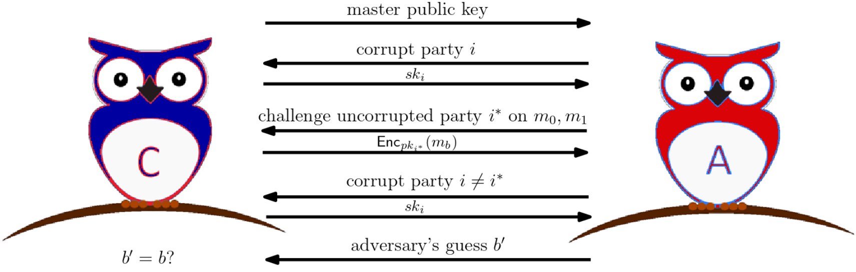
Introduction: Game-based Security



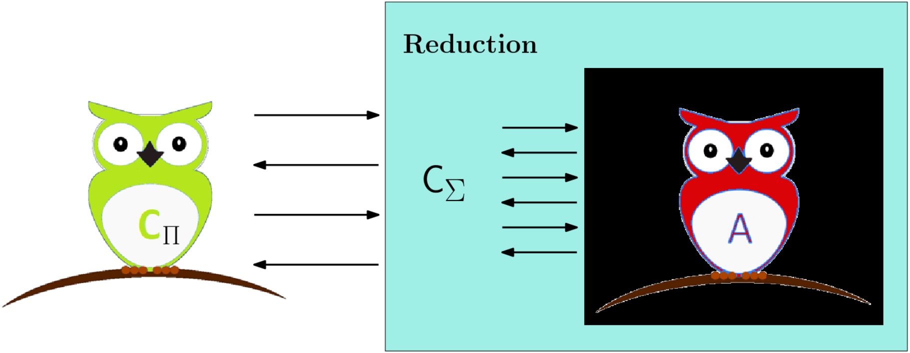
Introduction: Game-based Security

Identity-based Encryption



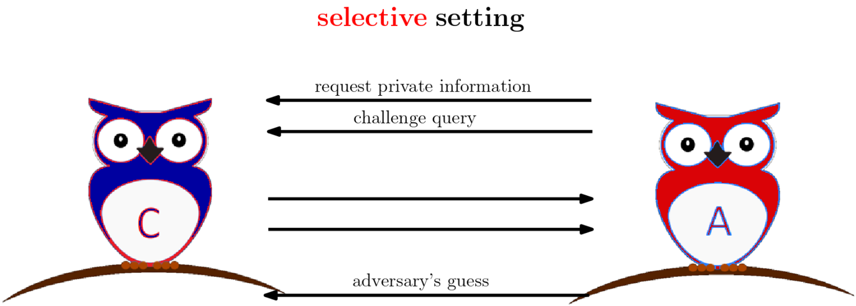
Introduction: Security Proof by Reduction

To prove **security of a scheme Σ**, relate it to some **hard problem Π**



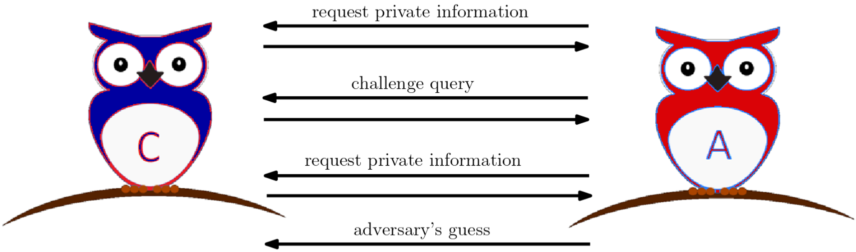
A **breaks Σ** with advantage $\epsilon \Rightarrow$ R **breaks Π** with advantage ϵ/loss

Introduction: Selective versus Adaptive Security



Introduction: Selective versus Adaptive Security

adaptive setting



This paper: Lower bounds on security loss against adaptive adversaries

Our Results

This paper: Lower bounds on security loss against adaptive adversaries

Consider certain multi-round games that capture several existing constructions where the adversary queries edges of a graph:

- **Generalized selective decryption (GSD)**:
  nodes = keys, edges = encryptions
- **TreeKEM construction of continuous group key agreement**:
  nodes = keys, sources = users, sinks = group keys, edges = encryptions
- **GGM84 construction of a prefix-constrained PRFs**:
  nodes = seeds, edges = PRG evaluations
- **Proxy re-encryption (PRE)**:
  nodes = keys, edges = re-encryption keys

Our Results

Application	Underlying Graph	Lower Bound	Reduction	Upper Bound
GSD	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FJP15]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [Pan07]
	Tree	$N^{\Omega(\log(N))}$	Straight-line	$N^{O(\log(N))}$ [FJP15]
	Arbitrary DAG	$2^{\Omega(\sqrt{N})}$	Oblivious	$N^{O(N/\log(N))}$ [JKK+17]
TreeKEM	Tree	$M^{\Omega(\log(\log(M)))}$	Straight-line	$Q^{O(\log(M))}$ [KPW+21]
GGM CPRF	Tree	$n^{\Omega(\log(n))}$	Straight-line	$n^{O(\log(n))}$ [FKPR14]
PRE	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Arbitrary DAG	$2^{\Omega(N)}$	Arbitrary	$N^{O(N/\log(N))}$ [FKKP19]

$N = 2^n$ ... size of the graph.
GGM CPRF: n ... input length. TreeKEM: M ... number of users, Q ... number of queries.

Reductions: oblivious $\subseteq$ straight-line $\subseteq$ arbitrary fully black-box

Our Results

Main conceptual idea:

- Introduce **Builder-Pebbler Game**:
  a two-player, multi-stage game
- Pebbler's success probability $\rightarrow$ lower bounds on security loss:
  use oracle separation techniques

Our Results

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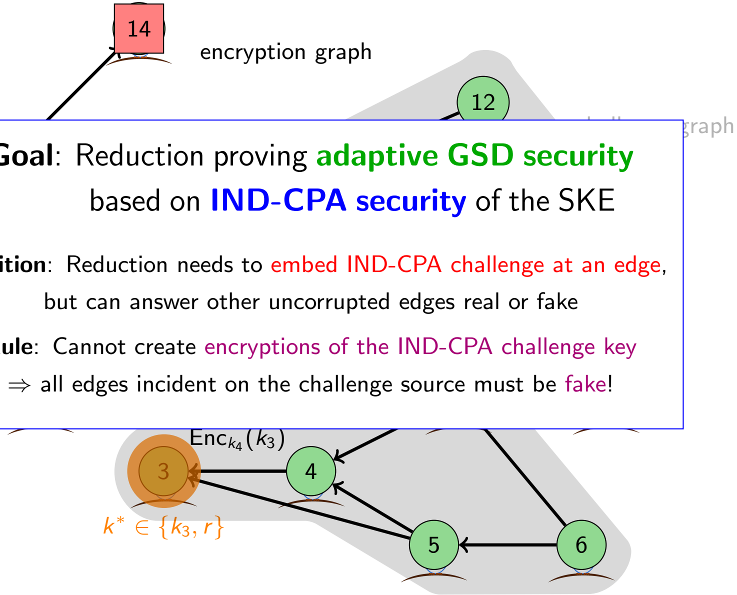
Generalized Selective Decryption (GSD) [Pan07]



Goal: Reduction proving **adaptive GSD security** based on **IND-CPA security** of the SKE

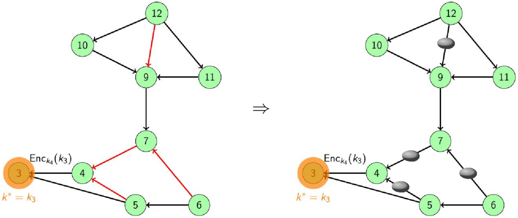
Intuition: Reduction needs to embed IND-CPA challenge at an edge, but can answer other uncorrupted edges real or fake

Rule: Cannot create encryptions of the IND-CPA challenge key $\Rightarrow$ all edges incident on the challenge source must be fake!

Threshold Adversaries

Our (inefficient) adversary:

- Corrupts all nodes outside the challenge graph, outputs 1 if any fake edges outgoing from corrupt nodes
  $\Rightarrow$ challenge key must be embedded in challenge graph
- On the challenge graph: Interprets **fake** edges as **pebbled**



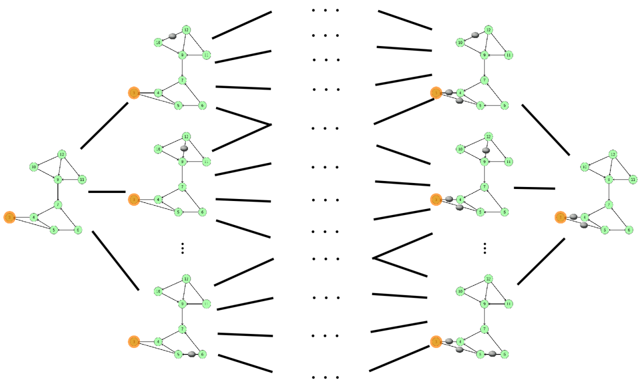
- Outputs 0/1 if final **pebbling configuration** good/bad

Threshold Adversaries

The threshold:

- Consider **reversible edge pebbling**:

  Can place/remove a pebble on an edge iff all edges incident on its source are pebbled.
- Define **good** by a cut in the configuration graph:



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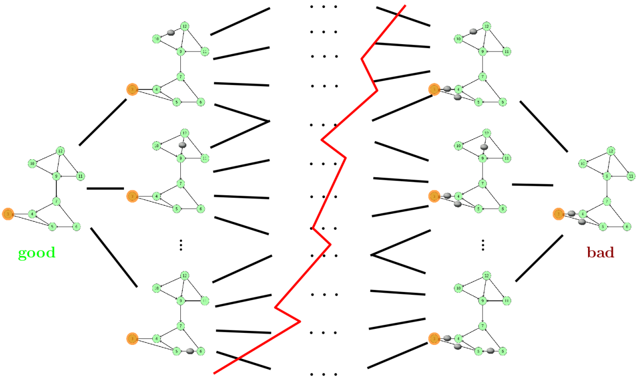

Cut set ... configurations at the border between good and bad

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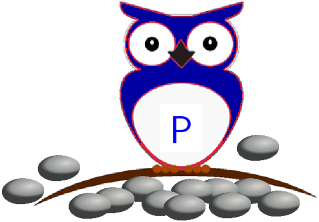
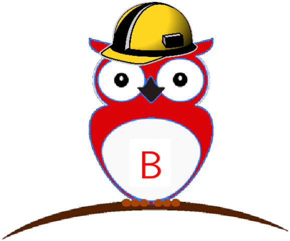
Builder-Pebbler Game

	1	2	3	4	5	6	7	8
Graph G	•	•	•	•	•	•	•	•
Cut set X								





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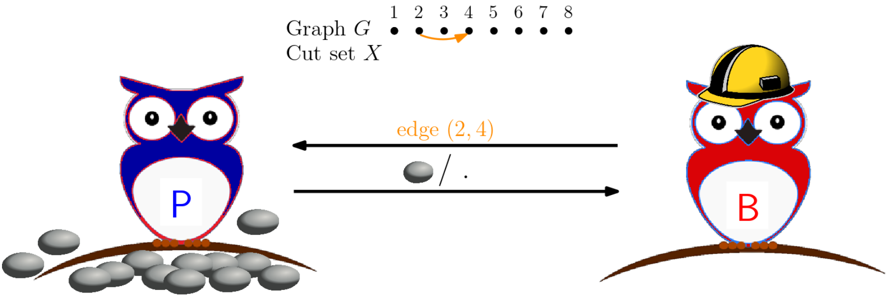

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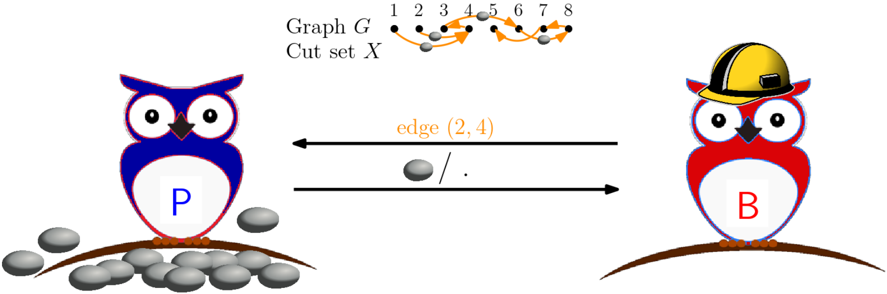
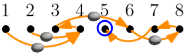
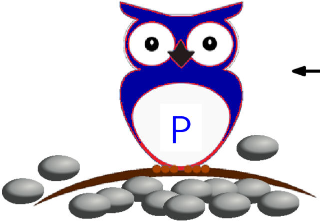


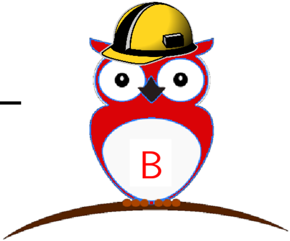
Builder-Pebbler Game

Graph G
Cut set X





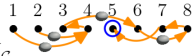
challenge node 5

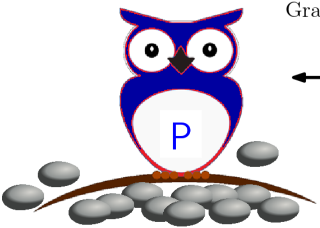


P wins iff $P(G) \in X$

Builder-Pebbler Game

Graph G
Cut set $X_{\mathcal{G}}$
Graph family $\mathcal{G}$





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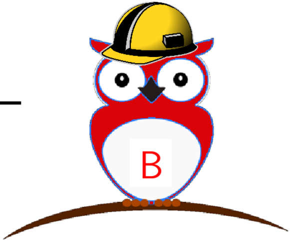
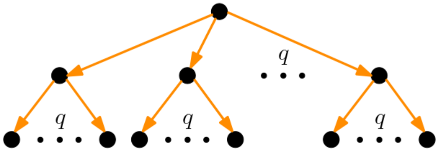
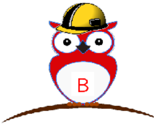


P wins iff $P(G) \in X_{\mathcal{G}}$
$\vee\, G \notin \mathcal{G}$

Builder Strategy for Trees





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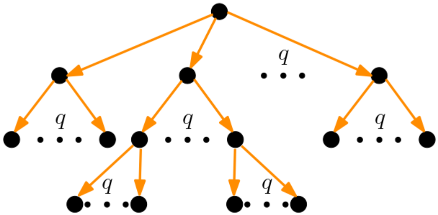

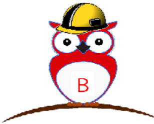

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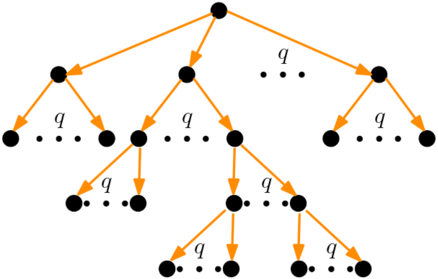

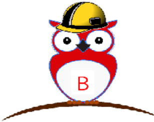

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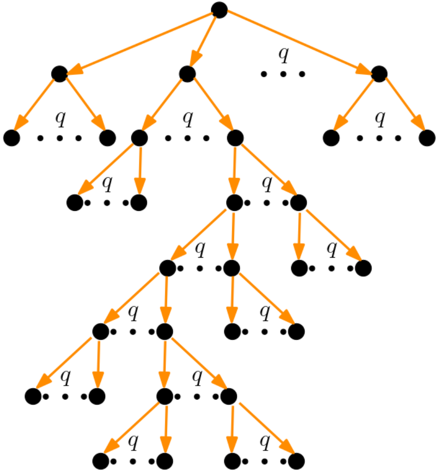

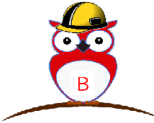

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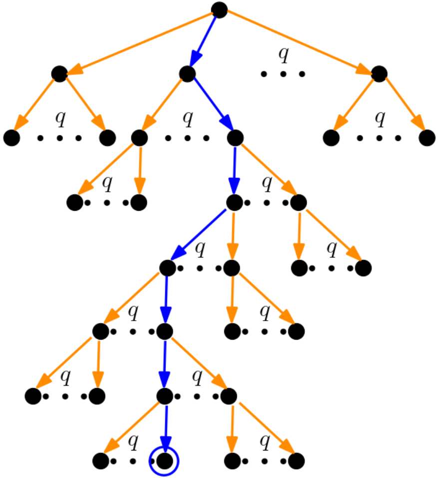
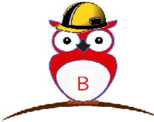


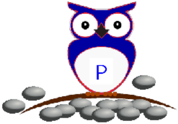


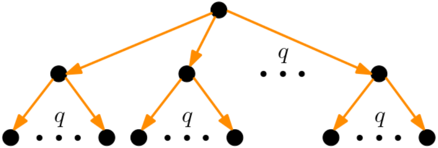
Cut for Trees with Large Outdegree

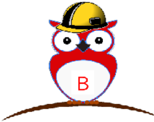
- Challenge graph = path of length n
- Lower bound for reversible edge pebbling on a path:
  Require $\log(n) + 1$ pebbles to pebble last edge
- Define **cut** X: pebble configuration P on the challenge path is good iff it is reachable with $\log(n)$ pebbles

$\Rightarrow$ **Goal of the Pebbler**: Place $\log(n)$ pebbles on the challenge path, but *no* pebbles outgoing from nodes outside the path.

Builder Strategy for Trees







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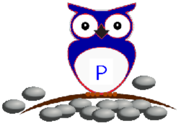

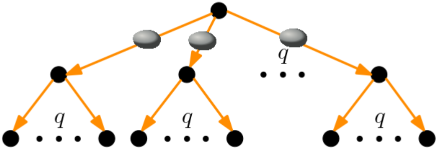

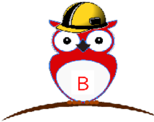

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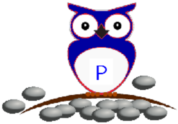

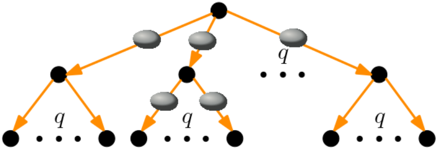

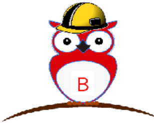

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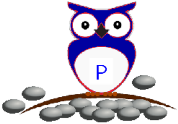

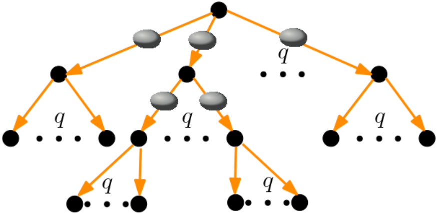

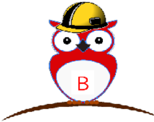

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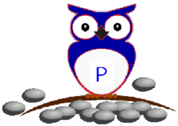

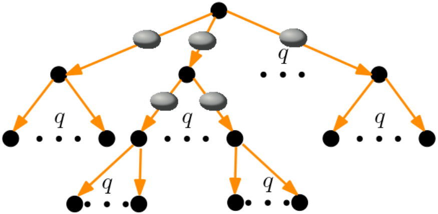

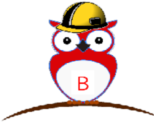

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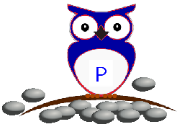

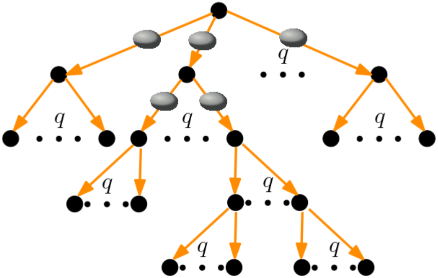

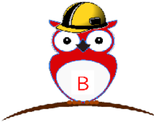

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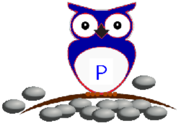

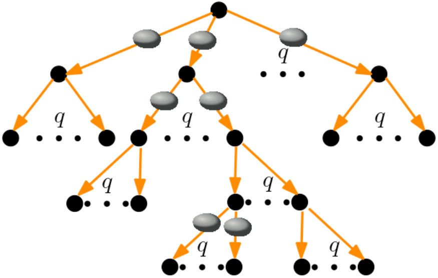

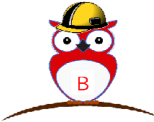

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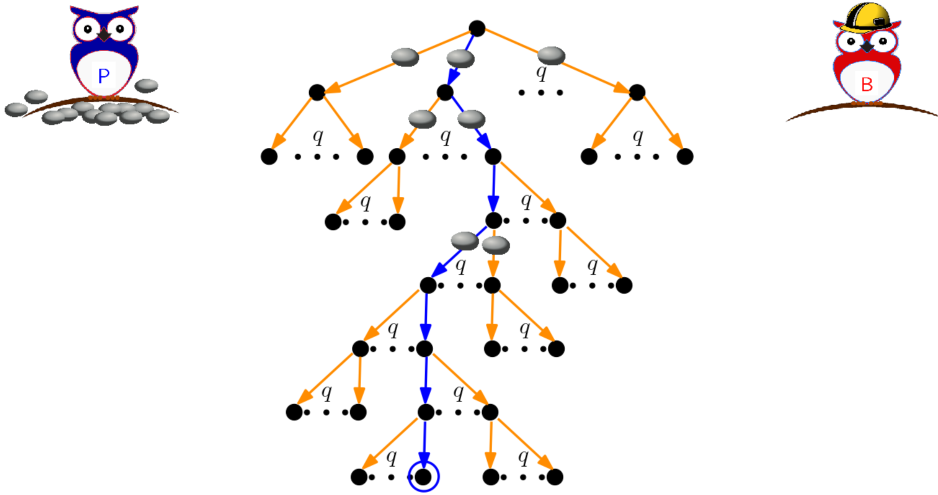

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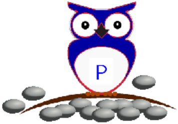
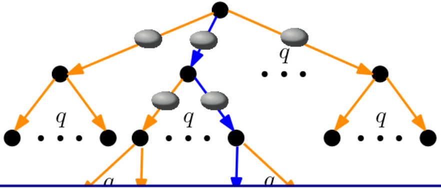
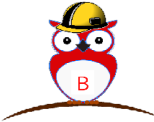
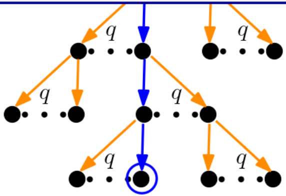






$$\text{Pebbler's success probability} \leq 1/q^{\log(n)-1} \underset{[q=O(n)]}{=} N^{-\Omega(\log(N))}$$



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Lower Bound for GSD

Combinatorial upper bound $\rightarrow$ cryptographic lower bound:

- Construct ideal SKE scheme
- Construct (inefficient) threshold adversary for GSD that simulates the above Builder strategy B, such that:

  $\forall$ straight-line reductions R: $\exists$ Pebbler P against B such that:

$$\text{R has security loss } \leq \Lambda \quad \Rightarrow \quad \text{P has advantage } \geq 1/\Lambda$$

Theorem (GSD on trees, informal)

Any **straight-line reduction** *proving security of unrestricted* **adaptive GSD** *based on the IND-CPA security of the underlying SKE scheme* **loses** *at least a* **super-polynomial** *factor ($N^{\Omega(\log(N))}$) in the number of users N.*

Our Results

Application	Underlying Graph	Lower Bound	Reduction	Upper Bound
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	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [Pan07]
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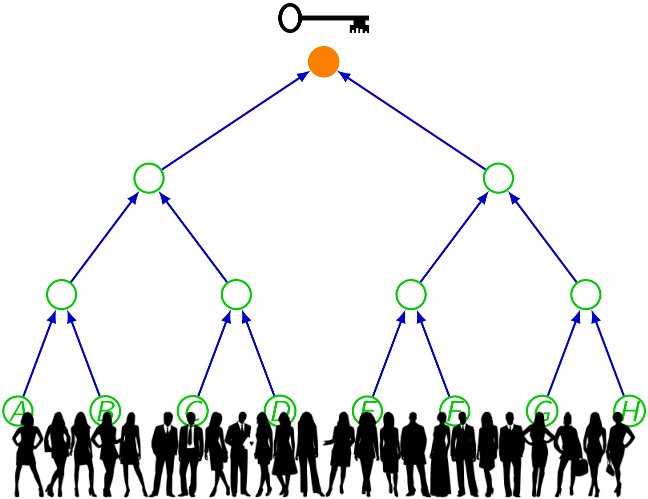
	Arbitrary DAG	$2^{\Omega(\sqrt{N})}$	Oblivious	$N^{O(N/\log(N))}$ [JKK+17]
TreeKEM	Tree	$M^{\Omega(\log(\log(M)))}$	Straight-line	$Q^{O(\log(M))}$ [KPW+21]
GGM CPRF	Tree	$n^{\Omega(\log(n))}$	Straight-line	$n^{O(\log(n))}$ [FKPR14]
PRE	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Arbitrary DAG	$2^{\Omega(N)}$	Arbitrary	$N^{O(N/\log(N))}$ [FKKP19]

$N = 2^n$ ... size of the graph.
GGM CPRF: n ... input length. TreeKEM: M ... number of users, Q ... number of queries.

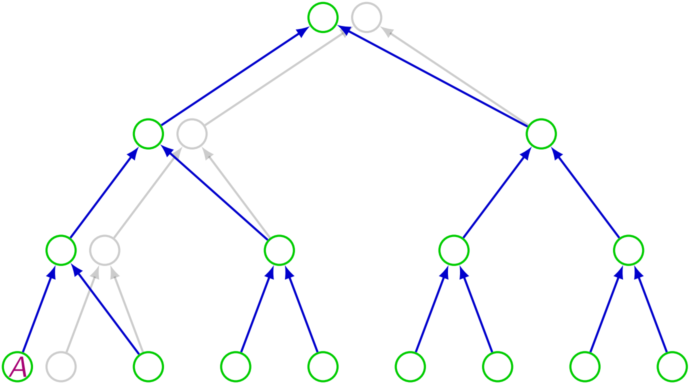
Continuous Group Key Agreement: TreeKEM [BBR18]



TreeKEM: Update

Alice updates:

- choose fresh keys (via hash chain, as in TreeKEM)
- remove old keys



Lower Bound for TreeKEM

- Game is quite similar to **public-key GSD**
- Construct adversary that embeds tree structure as above (depth $\log(M)$, M group size)

  Cruicial: Relay server is not trusted!

Theorem (TreeKEM, informal)

Any **straight-line reduction** *proving* **adaptive CGKA security for TreeKEM** *based on the IND-CPA security of the underlying PKE scheme* **loses** *a* **super-polynomial** *factor* $(M^{\Omega(\log\log(M))})$ *in the group size* M.

Our Results

Application	Underlying Graph	Lower Bound	Reduction	Upper Bound
GSD	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FJP15]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [Pan07]
	Tree	$N^{\Omega(\log(N))}$	Straight-line	$N^{O(\log(N))}$ [FJP15]
	Arbitrary DAG	$2^{\Omega(\sqrt{N})}$	Oblivious	$N^{O(N/\log(N))}$ [JKK+17]
TreeKEM	Tree	$M^{\Omega(\log(\log(M)))}$	Straight-line	$Q^{O(\log(M))}$ [KPW+21]
GGM CPRF	Tree	$n^{\Omega(\log(n))}$	Straight-line	$n^{O(\log(n))}$ [FKPR14]
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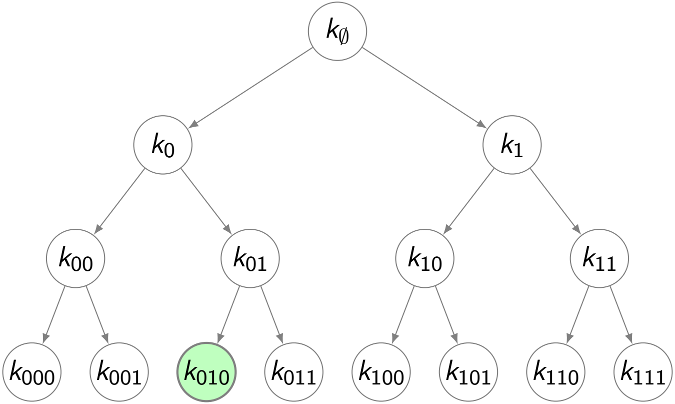
	Arbitrary DAG	$2^{\Omega(N)}$	Arbitrary	$N^{O(N/\log(N))}$ [FKKP19]

$N = 2^n$ ... size of the graph.
GGM CPRF: n ... input length. TreeKEM: M ... number of users, Q ... number of queries.

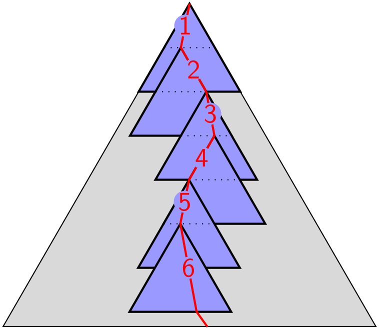
Prefix-constrained PRF: GGM84



$$\mathsf{F}_{GGM}(k, x) = k_x \text{ where } k_\emptyset = k \text{ and } \forall z \in \{0,1\}^* : k_{z\|0} \| k_{z\|1} = \mathsf{PRG}(k_z)$$

Adversary can query constrained keys and evaluations.

Lower Bound for GGM84



Theorem (GGM CPRF, informal)

Any **straight-line reduction** *proving* **adaptive security for the GGM CPRF** *based on the security of the underlying PRG* **loses** *a* **super-polynomial** *factor ($n^{\Omega(\log(n))}$) in the input size n.*

Our Results

Application	Underlying Graph	Lower Bound	Reduction	Upper Bound
GSD	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FJP15]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [Pan07]
	Tree	$N^{\Omega(\log(N))}$	Straight-line	$N^{O(\log(N))}$ [FJP15]
	Arbitrary DAG	$2^{\Omega(\sqrt{N})}$	Oblivious	$N^{O(N/\log(N))}$ [JKK+17]
TreeKEM	Tree	$M^{\Omega(\log(\log(M)))}$	Straight-line	$Q^{O(\log(M))}$ [KPW+21]
GGM CPRF	Tree	$n^{\Omega(\log(n))}$	Straight-line	$n^{O(\log(n))}$ [FKPR14]
PRE	Path P_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Binary In-Tree B_N	$N^{\Omega(\log(N))}$	Oblivious	$N^{O(\log(N))}$ [FKKP19]
	Arbitrary DAG	$2^{\Omega(N)}$	Arbitrary	$N^{O(N/\log(N))}$ [FKKP19]

$N = 2^n$ . . . size of the graph.
GGM CPRF: n . . . input length. TreeKEM: M . . . number of users, Q . . . number of queries.

For the other results, see https://eprint.iacr.org/2021/059!

Table of Contents

Conclusion and Open Problems

Initiated study of **lower bounds** on loss in **adaptive security** for certain **multi-round** games on graphs.

- Can we strengthen our lower bounds to hold also for rewinding / non-obliviousness reductions? Or can we use these techniques to overcome our lower bounds?

  PRE on complete DAGs: LB for arbitrary black-box reductions.

- What are other multi-round games captured by the Builder-Pebbler Game?
- Can we use pebbling lower bounds to prove lower bounds on the loss in adaptive security in other settings, i.e. constant-round games (eg. ABE, Garbling)?

  Yao's garbling: Yes [KKPW21], but very different techniques required

THANK YOU FOR YOUR ATTENTION!